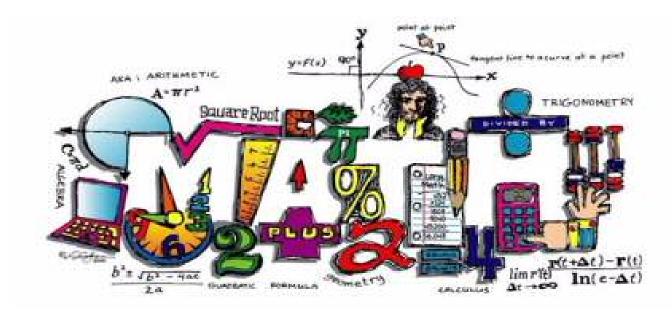
Period

# **Geometry Honors Incoming Assignment**

\*\*\*\*

**Bak MSOA Summer Required Mathematics Assignment Directions:** 



Complete the problems on each page.

Show all appropriate work and circle your answers.

The work will not be collected on the first day of school.

This will be a part of your first nine weeks Assignment grade.

GOOD LUCK WITH YOUR ASSIGNMENT!

WE LOOK FORWARD TO SEEING YOU IN AUGUST. 

©

# Reteach Translations in the Coordinate Plane

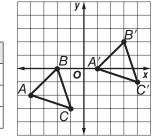
A translation is the movement of a geometric figure in some direction without turning the figure. When translating a figure, every point of the original figure is moved the same distance and in the same direction. To graph a translation of a figure, move each vertex of the figure in the given direction. Then connect the new vertices.

Example

Triangle ABC has vertices A(-4, -2), B(-2, 0), and C(-1, -3). Find the vertices of triangle A'B'C' after a translation of 5 units right and 2 units up.

Add 5 to each *x*-coordinate. Add 2 to each *y*-coordinate.

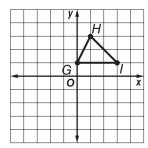
Vertices of $\triangle ABC$	(x+5,y+2)	Vertices of $\triangle A'B'C'$
A(-4, -2)	(-4+5, -2+2)	A'(1, 0)
B(-2, 0)	(-2+5,0+2)	B'(3, 2)
C(-1, -3)	(-1+5, -3+2)	C'(4, -1)



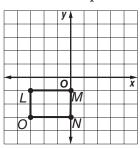
The coordinates of the vertices of  $\triangle A'B'C'$  are A'(1, 0), B'(3, 2), and C'(4, -1).

#### Exercises

**1.** Translate  $\triangle GHI$  1 unit left and 5 units down.

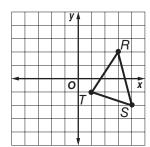


2. Translate rectangle LMNO 4 units right and 3 units up.

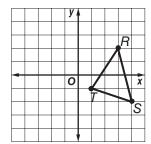


Triangle RST has vertices R(3, 2), S(4, -2), and T(1, -1). Find the vertices of R'S'T' after each translation. Then graph the figure and its translated image.

3. 5 units left, 1 unit up



4. 3 units left, 2 units down

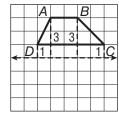


### Reflections in the Coordinate Plane

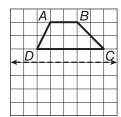
The mirror image produced by flipping a figure over a line is called a **reflection**. This line is called the line of reflection. A reflection is one type of transformation or mapping of a geometric figure. In mathematics, an **image** is the position of a figure after a transformation. The image of point A is written A'. A' is read as A prime.

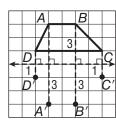
#### Example

Draw the image of quadrilateral ABCD after a reflection over the given line.

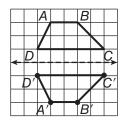


Step 1 Count the number of units between each vertex and the line of reflection.





**Step 2** To find the corresponding point for vertex A, move along the line through vertex A perpendicular to the line of reflection until you are 3 units from the line on the opposite side. Draw a point and label it A'. Repeat for each vertex.

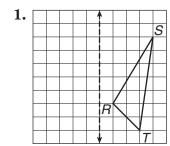


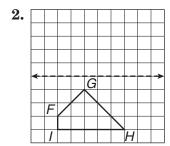
**Step 3** Connect the new vertices to form quadrilateral A'B'C'D'.

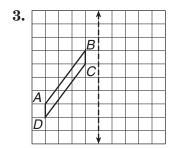
Notice that if you move along quadrilateral ABCD from A to B to C to D, you are moving in the clockwise direction. However, if you move along quadrilateral A'B'C'D' from A' to B' to C' to D', you are moving in the counterclockwise direction. A figure and its reflection have opposite orientations.

### Exercises

Draw the image of the figure after a reflection over the given line.







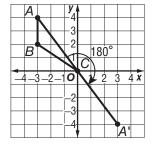
# Rotations in the Coordinate Plane

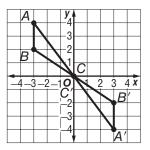
- A rotation occurs when a figure is rotated around a point.
- Another name for a rotation is a turn.
- In a clockwise rotation of 90° about the origin, the point (x, y) becomes (y, -x).
- In a clockwise rotation of 180° about the origin, the point (x, y) becomes (-x, -y).
- In a clockwise rotation of 270° about the origin, the point (x, y) becomes (-y, x).

Example

Triangle ABC has vertices A(-3, 4), B(-3, 2), C(0, 0). Rotate triangle ABC clockwise 180° about the origin.

- Step 1 Graph triangle *ABC* on a coordinate plane.
- Step 2 Sketch segment *AO* connecting point *A* to the origin. Sketch another segment *A'O* so that the angle between points A, O, and A' measures 180° and the segment is congruent to AO.
- Step 3 Repeat for point B (point C won't move since it is at the origin). Then connect the vertices to form triangle A'B'C'.





### Exercises

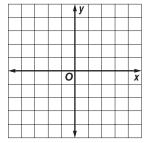
Find the coordinates of the image of (2, 4), (1, 5), (1, -3) and (3, -4) under each transformation.

- **1.** a clockwise rotation of 90° about the origin
- **2.** a clockwise rotation of 270° about the origin

 $\triangle RST$  has vertices R(-2, 1), S(3, 3), and T(0, 0). Graph the figure and its image after each rotation. Then give the coordinates of the vertices for triangle R'S'T'.

- **3.** 180° counterclockwise about the origin
  - o

4. 90° counterclockwise about the origin



### Dilations

The image produced by enlarging or reducing a figure is called a **dilation**.

**Example 1** Graph  $\triangle ABC$  with vertices A(-2, -1), B(2, 3), and C(2, -1). Then graph its image  $\triangle A'B'C'$  after a dilation with

$$C(2,-1) \qquad \rightarrow \qquad \left(2 \cdot \frac{3}{2}, -1 \cdot \frac{3}{2}\right) \qquad \rightarrow \qquad C'\left(3, -1\frac{1}{2}\right)$$

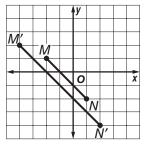


**Example 2** Segment M'N' is a dilation of segment MN. Find the scale factor of the dilation, and classify it as an enlargement or a reduction.

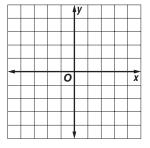
Write the ratio of the *x*- or *y*-coordinate of one vertex of the dilated figure to the x- or y-coordinate of the corresponding vertex of the original figure. Use the *x*-coordinates of N(1, -2)and N'(2, -4).

$$\frac{x\text{-coordinate of point }N'}{x\text{-coordinate of point }N} = \frac{2}{1} \text{ or } 2$$

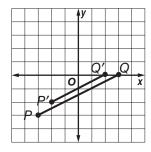
The scale factor is 2. Since the image is larger than the original figure, the dilation is an enlargement.



**1.** Polygon *ABCD* has vertices A(2, 4), B(-1, 5), C(-3, -5), and D(3, -4). Find the coordinates of its image after a dilation with a scale factor of  $\frac{1}{2}$ . Then graph polygon *ABCD* and its dilation.



**2.** Segment P'Q' is a dilation of segment PQ. Find the scale factor of the dilation, and classify it as an enlargement or a reduction.

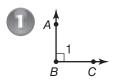


### Classify Angles

- An angle is formed by two rays that share a common endpoint called the vertex.
- An angle can be named in several ways. The symbol for angle is ∠.
- · Angles are classified according to their measure. Two angles that have the same measure are said to be congruent.
- Two angles are vertical if they are opposite angles formed by the intersection of two lines. Vertical angles are congruent.
- Two angles are adjacent if they share a common vertex, a common side, and do not overlap.

**Acute Angle Right Angle Obtuse Angle Straight Angle** exactly 90° less than 90° between 90° and 180° exactly 180°

Examples Name each angle below. Then classify the angle as acute, right, obtuse, or straight.



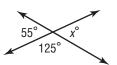
Use the vertex as the middle letter and a point from each side,  $\angle ABC$ ,  $\angle CBA$ , or use the vertex or the number only,  $\angle B$  or  $\angle 1$ . The angle is 90°, so it is a right angle.



Use the vertex or the number only,  $\angle D$  or  $\angle 2$ . The angle is less than 90°, so it is an acute angle.



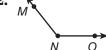
What is the value of x in the figure at the right? The angle labeled  $x^{\circ}$  and the angle labeled 55° are vertical angles. Since vertical angles are congruent, the value of x is 55.



#### Exercises

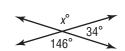
Name each angle. Then classify the angle as acute, right, obtuse, or straight.







**4.** Find the value of *x* in the figure at the right.

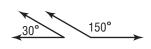


## Complementary and Supplementary Angles

- Two angles are **complementary** if the sum of their measures is 90°.
- Two angles are **supplementary** if the sum of their measures is 180°.

**Examples** Identify each pair of angles as *complementary*, supplementary, or neither.

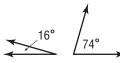




$$30^{\circ} + 150^{\circ} = 180^{\circ}$$

The angles are supplementary.





$$16^{\circ} + 74^{\circ} = 90^{\circ}$$

The angles are complementary.

#### Example 3

ALGEBRA Find the value of x.

Since the two angles are supplementary, the sum of their measures is 180°.



$$x + 35 = 180$$

Write the equation.

$$-35 -35$$

Subtract 35 from each side.

$$x = 145$$

Simplify.

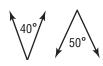
So, the value of x is 145.

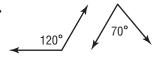


#### Exercises

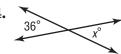
Identify each pair of angles as complementary, supplementary, or neither.







ALGEBRA Find the value of x in each figure.





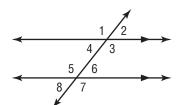


Lines

- Perpendicular lines are lines that intersect at right angles.
- Parallel lines are two lines in a plane that never intersect or cross.
- A line that intersects two or more other lines is called a transversal.
- If the two lines cut by a transversal are parallel, then these are special pairs of angles are congruent: alternate interior angles, alternate exterior angles, and corresponding angles.

Example 1

Classify  $\angle 4$  and  $\angle 8$  as alternate interior, alternate exterior, or corresponding.



 $\angle 4$  and  $\angle 8$  are in the same position in relation to the transversal on the two lines. They are corresponding angles.

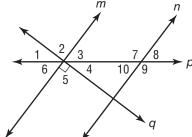
**Example 2** Refer to the figure in Example 1. Find  $m \angle 2$  if  $m \angle 8 = 58^\circ$ .

Since  $\angle 2$  and  $\angle 8$  are alternate exterior angles,  $m\angle 2 = 58^{\circ}$ 

Exercises

In the figure at the right, lines m and line n are parallel. If  $m \angle 3 = 64^{\circ}$ , find each given angle measure. Justify each answer.





- **2.** *m*∠10
- 3.  $m \angle 4$
- **4.** *m*∠6



# **Triangles**

- A triangle is formed by three line segments that intersect only at their endpoints.
- A point where the segments intersect is a **vertex** of the triangle.
- Every triangle also has three angles. The sum of the measure of the angles is 180°.
- All triangles have at least two acute angles. Triangles can be classfied by the measure of its third angle: acute, right, or obtuse.
- Another way to classify triangles is by their sides: scalene, isosceles, or equilateral.

#### **Example 1** Find the value of x in $\triangle ABC$ .

$$x + 66 + 52 = 180$$

The sum of the measures is 180.

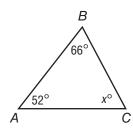
$$x + 118 = 180$$

Simplify.

$$-118 - 118$$

Subtract 118 from each side.

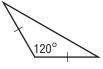
$$x = 62$$



The value of x is 62.

### **Example 2** Classify the triangle by its angles and by its sides.

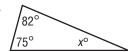
The triangle has one obtuse angle and two sides the same length. So, it is an obtuse, isosceles triangle.

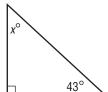


#### **Exercises**

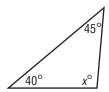
Find the the value of x in each triangle. Then classify the triangle as acute, right, or obtuse.

1.



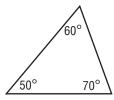


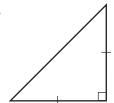
3.

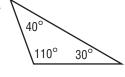


Classify each triangle by its angles and by its sides.

4.

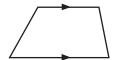






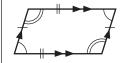
### Quadrilaterals

- A quadrilateral is a closed figure with four sides and four angles.
- · Quadrilaterals are named based on their sides and angles.



**Trapezoid** 

quadrilateral with exactly one pair of parallel sides



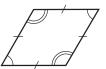
**Parallelogram** 

quadrilateral with opposite sides parallel and opposite sides congruent



Rectangle

parallelogram with 4 right angles



Rhombus

parallelogram with 4 congruent sides

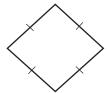


Square

parallelogram with 4 right angles and 4 congruent sides

#### **Examples**









The quadrilateral is a parallelogram with 4 congruent sides. It is a rhombus.

The quadrilateral has one pair of parallel sides. It is a trapeziod.

**Example 3** Find the value of x in the quadrilateral shown.

$$123 + 90 + 74 + x = 360$$

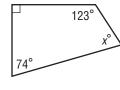
$$287 + x = 360$$

$$-287 = -287$$

$$x = 73$$

Write the equation. Simplify.

Subtract.



So, the value of x is 73.

### Exercises

Classify the quadrilateral using the name that best describes it.







### Polygons and Angles

- A polygon is a simple, closed figure formed by three or more line segments. The segments intersect only at their endpoints.
- Polygons can be classified by the number of sides they have.
- The sum of the measures of the interior angles of a polygon is (n-2)180, where n represents the number of sides.

#### Example 1

Determine whether the figure is a polygon. If it is, classify the polygon. If it is not a polygon, explain why.

The figure has 8 sides that only intersect at their endpoints. It is an octagon.



#### Example 2

The defense department of the United States has its headquarters in a building called the Pentagon because it is shaped like a regular pentagon. Find the measure of an interior angle of a regular pentagon.

S = (n-2)180Write an equation.

S = (5 - 2)180Replace *n* with 5. Subtract.

S = (3)180Multiply.

S = 540The sum of the interior angles is 540°.

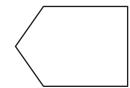
 $540 \div 5 = 108$ Divide by the number of interior angles to find the measure of one angle.

The measure of one interior angle of a regular pentagon is 108°.

#### Exercises

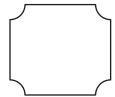
Determine whether the figure is a polygon. If it is, classify the polygon. If it is not a polygon, explain why.

1.





3.



Find the sum of the interior angle measures of each polygon.

**4.** nonagon (9-sided)

**5.** 14-gon

Find the measure of one interior angle in each regular polygon.

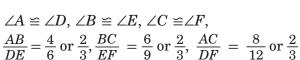
6. hexagon

**7.** 15-gon

### Similar Polygons

Two polygons are similar if they have the same shape. If the polygons are similar, then their corresponding angles are congruent and the measures of their corresponding sides are proportional. Use the symbol  $\sim$  for similarity.

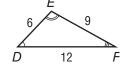
**Example 1** Determine whether  $\triangle ABC$  is similar to  $\triangle DEF$ . Explain.



The corresponding angles are congruent, and the corresponding sides are proportional.

So,  $\triangle ABC$  is similar to  $\triangle DEF$ , or  $\triangle ABC \sim \triangle DEF$ .





### Example 2

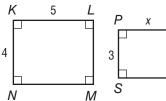
Given that polygon  $KLMN \sim \text{polygon } PQRS$ , find the missing

Find the scale factor from polygon *KLMN* to polygon *PQRS*.

$$\mbox{scale factor:} \frac{PS}{KN} = \frac{3}{4} \qquad \mbox{The scale factor is the} \\ \mbox{constant of proportionality}.$$

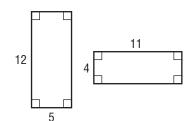
A length on polygon PQRS is  $\frac{3}{4}$  times as long as a corresponding length on polygon KLMN.

$$x=rac{3}{4}\,(5)$$
 Write the equation.  $x=rac{15}{4}\,\mathrm{or}\,\,3.75$  Multiply.

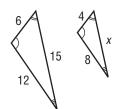


#### Exercises

**1.** Determine whether the polygons below are similar. Explain.



**2.** The triangles below are similar. Find the missing measure.

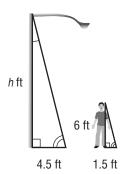


### **Indirect Measurement**

Indirect measurement allows you to use properties of similar polygons to find distances or lengths that are difficult to measure directly.

Example

LIGHTING Tyrone is standing next to a lightpole in the middle of the day. Tyrone's shadow is 1.5 feet long, and the lightpole's shadow is 4.5 feet long. If Tyrone is 6 feet tall, how tall is the lightpole?



Write a proportion and solve.

Tyrone's shadow 
$$\rightarrow$$
 lightpole's shadow  $\rightarrow$ 

$$\frac{1.5}{4.5} = \frac{6}{h}$$

$$\leftarrow \mathsf{Tyrone's}\;\mathsf{height}$$

$$\leftarrow \text{lightpole's height}$$

$$1.5 \cdot h = 4.5 \cdot 6$$

Find the cross products.

$$1.5h = 27$$

Multiply.

Simplify.

$$\frac{1.5h}{1.5} = \frac{27}{1.5}$$

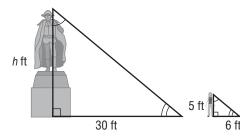
Division Property of Equality

$$a = 18$$

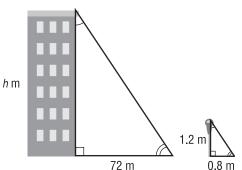
The lightpole is 18 feet tall.

#### Exercises

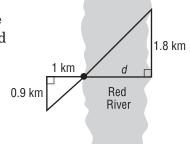
1. MONUMENTS A statue casts a shadow 30 feet long. At the same time, a person who is 5 feet tall casts a shadow that is 6 feet long. How tall is the statue?



**2. BUILDINGS** A building casts a shadow 72 meters long. At the same time, a parking meter that is 1.2 meters tall casts a shadow that is 0.8 meter long. How tall is the building?



3. SURVEYING The two triangles shown in the figure are similar. Find the distance d across Red River.



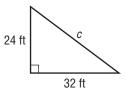
## The Pythagorean Theorem

The Pythagorean Theorem describes the relationship between the lengths of the legs and the hypotenuse for any right triangle. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. You can use the Pythagorean Theorem to find the length of a side of a right triangle if the lengths of the other two sides are known.

Examples

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.





$$a^2 + b^2 = c^2$$

$$24^2 + 32^2 = c^2$$

$$576 + 1,024 = c^2$$

$$\frac{1,600}{\pm\sqrt{1,600}} = c^2 \\
\pm\sqrt{1,600} = c$$

$$c = 40 \text{ or } -40$$



$$a^2 + b^2 = c^2$$

$$15^2 + b^2 = 20^2$$

$$225 + b^2 = 400$$

$$225 + b^2 - 225 = 400 - 225$$
$$b^2 = 175$$

$$\sqrt{b^2} = \pm \sqrt{175}$$

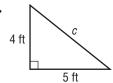
$$b \approx \pm 13.2$$

Length must be positive, so the length of the hypotenuse is 40 feet. The length of the other leg is about 13.2 centimeters.

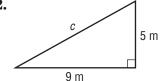
#### Exercises

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Round to the nearest tenth if necessary.

1.



2.





**4.** 
$$a = 7$$
 km,  $b = 12$  km

**5.** 
$$a = 10$$
 yd,  $c = 25$  yd

**6.** 
$$b = 14$$
 ft,  $c = 20$  ft

## Use the Pythagorean Theorem

The Pythagorean Theorem can be used to solve a variety of problems.

#### Example

A professional ice hockey rink is 200 feet long and 85 feet wide. What is the length of the diagonal of the rink?



The Pythagorean Theorem

$$200^2 + 85^2 = c^2$$

Replace a with 200 and b with 85.

$$40,000 + 7,225 = c^2$$

Evaluate 200<sup>2</sup> and 85<sup>2</sup>.

$$47,225 = c^2$$

Add 40,000 and 7,225.

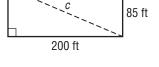
$$\sqrt{47,225} = c^2$$

Definition of square root

$$\sqrt{217.3} \approx c$$

Use a calculator.

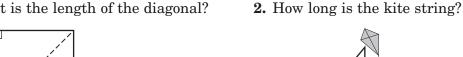
The length of the diagonal of an ice hockey rink is about 217.3 feet.

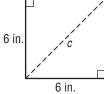


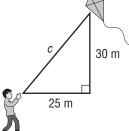
#### **Exercises**

Write an equation that can be used to answer the question. Then solve. Round to the nearest tenth if necessary.

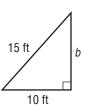
**1.** What is the length of the diagonal?



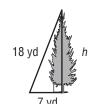




**3.** What is the height of the ramp?



**4.** How tall is the tree?



### Distance on the Coordinate Plane

You can use the Pythagorean Theorem to find the distance between two points on the coordinate plane.

#### Example

Graph the ordered pairs (2, -3) and (5, 4). Then find the distance c between the two points.

$$a^2 + b^2 = c^2$$

The Pythagorean Theorem

$$3^2 + 7^2 = c^2$$

Replace a with 3 and b with 7.

$$58 = c^2$$

$$+\sqrt{58} = \sqrt{c^2}$$

$$3^2 + 7^2 = 9 + 49$$
, or 58.

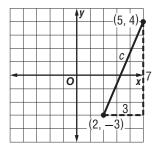
$$\pm \sqrt{58} = \sqrt{c^2}$$

Definition of square root

$$\pm 7.6 \approx c$$

Use a calculator.

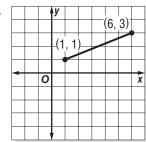
The points are about 7.6 units apart.



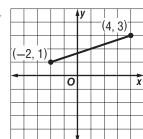
#### Exercises

Find the distance between each pair of points whose coordinates are given. Round to the nearest tenth if necessary.

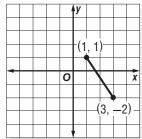
1.



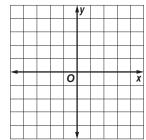
2.



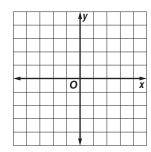
3.



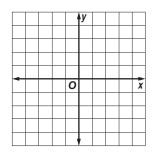
Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth if necessary.



**5.** 
$$(0, -4), (-3, 0)$$



**6.** 
$$(-1, 1), (-4, 4)$$



45°

# Reteach

## Special Right Triangles

Example 1

Triangle ABC and triangle UVW are  $45^{\circ}-45^{\circ}-90^{\circ}$  triangles. Find the length of the hypotenuse in  $\Delta UVW$ .

The scale factor from  $\triangle ABC$  to  $\triangle UVW$  is  $\frac{3}{1}$ 

or 3. Use the scale factor to find the hypotenuse.

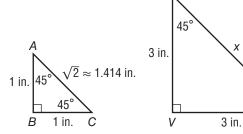
$$x = 3 \cdot \sqrt{2}$$

Multiply the length of  $\overline{AC}$ 

$$=3\sqrt{2}$$

by the scale factor, 3.

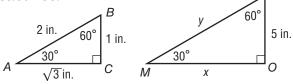
So, the hypotenuse of  $\Delta UVW$  measures  $3\sqrt{2}$  inches.



Example 2

Triangle ABC and triangle MNO are 30°-60°-90° triangles. Find the exact length of the missing measures.

The scale factor from  $\triangle ABC$  to  $\triangle MNO$  is 5. Use the scale factor to find the missing measures.



 $y = 5 \cdot 2 \text{ or } 10$ 

Multiply the length  $\overline{AB}$  by the scale factor.

So, *y* is 10 inches.

 $x = 5 \cdot \sqrt{3} \text{ or } 5\sqrt{3}$ 

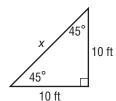
Multiply the length of  $\overline{AC}$  by the scale factor.

So, x is  $5\sqrt{3}$  inches.

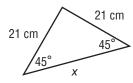
### **Exercises**

Find each missing measure.

1.



2.



3.

